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Berry phase in Tavis-Cummings model

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Abstract. In this paper we investigate the Berry phase in Tavis-Cummings model in the rotating wave approximation. The dipole-dipole interaction between the atoms is considered. The eigenfunctions of the system are obtained and thus the Berry phase is evaluated explicitly in terms of the introduction of the phase shift. It is shown that the Berry phase can be easily controlled by the atom-cavity coupling strength, the cavity frequency detuning, which can be important in applications in geometric quantum computing.

PACS. 03.65.Vf Phases: geometric; dynamic or topological – 03.65.Yz Decoherence; open systems; quantum statistical methods

1 Introduction

The Berry phase [1], which is an important topic in modern physics, describes a novel phase factor of the wave functions depending only on the geometry of the path when a time-dependent quantum system undergoes an adiabatic and cyclic evolution. Recently, the Berry phase has been regarded as an essential way to implement operation of a universal quantum logic gate in quantum computing. Cavity QED is an important solid-state system for processing quantum information and implementing quantum computing [2]. It is shown that the Berry phase can be easily controlled by the coupling strength and the frequency detuning of the cavity field [3–9,16], which can be important in applications in geometric quantum computing [3,4].

It should be noticed that above investigations are limited to the framework of the classical controlled external field, namely, this external field itself has not been quantized. It is known in quantum optics that a quantized field can lead to many novel quantum effects such as quantum jumps, collapses and revivals of the Rabi oscillations. Moreover, if the quantum system interacts with the vacuum, spontaneous emission and Lamb shift can also be observed in experiment. Latterly, a novel Berry phase, which has no zero value in the vacuum state, can also be induced if the quantized field is controlled adiabatically and periodically [11,17]. However, this investigation so far has been restricted to single qubit Jaynes-Cummings model. In this paper, we extend this method to Tavis-Cummings model [12] that there are two two-level atoms coupling with a single mode quantized field cavity, and the dipole-dipole interaction between atoms is turned on. Yuan and Zhu [13] investigated two coupled quantum dots which are embedded in a high-Q single mode cavity and coupled to the common phonon fields, and discussed the influence of the environmental temperatures on the Berry phase. In this paper we study the novel behavior of the Berry phase of the ground state. The eigen-functions of the system are obtained and thus the Berry phase is evaluated explicitly in terms of the introduction of the phase shift.

The dipole-dipole interaction of the atoms can not be neglected when the distance of the atoms is less than the wave length in the cavity. The Hamiltonian can be given in the rotating wave approximation [14]

$$H = \omega a^\dagger a + \frac{\omega_0}{2}\left(\sigma_1^z + \sigma_2^z\right) + \lambda\left[a\left(\sigma_1^+ + \sigma_2^+\right) + a^\dagger\left(\sigma_1^- + \sigma_2^-\right)\right] + J\left(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+\right), \quad (1)$$

where Pauli operator $\sigma_i^z = |e\rangle_i\langle e| - |g\rangle_i\langle g|$, $\sigma_i^+ = |e\rangle_i\langle g|$ and $\sigma_i^- = |g\rangle_i\langle e|$, with $|e\rangle_i$ and $|g\rangle_i$ being the excited and ground states of the ith atom, $i = 1, 2$, $a^\dagger$and a are the creation and annihilation operators for the cavity mode, ω_0 is the atomic transition frequency, ω is the cavity frequency, λ is the atom-cavity coupling strength and J is dipole-dipole coupling strength between atoms. In the computation vectors of the Hilbert space such that $|n, e_1, e_2\rangle$, $|n+1, e_1, g_2\rangle$, $|n+1, g_1, e_2\rangle$ and $|n+2, g_1, g_2\rangle$, the matrix of Hamiltonian (1) is given by (in an appropriate interaction picture)

$$H = \begin{pmatrix} \Delta & \lambda\sqrt{n+1} & \lambda\sqrt{n+1} & 0 \\ \lambda\sqrt{n+1} & 0 & 2J & \lambda\sqrt{n+2} \\ \lambda\sqrt{n+1} & 2J & 0 & \lambda\sqrt{n+2} \\ 0 & \lambda\sqrt{n+2} & \lambda\sqrt{n+2} & -\Delta \end{pmatrix}. \quad (2)$$

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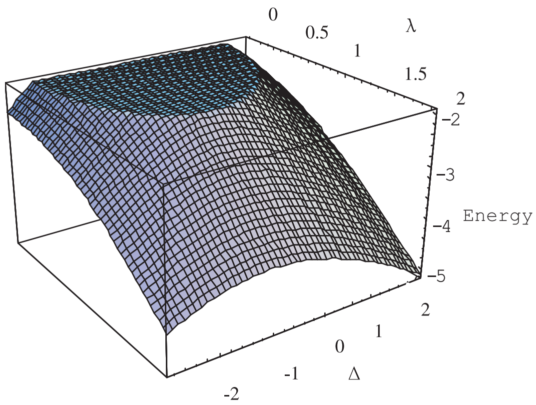

Fig. 1. The ground-state energy versus the coupling strength λ and the cavity detuning Δ. Δ and λ are measured in units of J.

where $\Delta = \omega_0 - \omega$ is frequency detuning of the cavity field. The instantaneous eigenvalues $E_j (j = 1, 2, 3, 4)$ and the corresponding eigenstates $|\psi_j\rangle$ of Hamiltonian (2) can be found analytically; however, it is pointless to display the tedious formula in the present paper. For simplicity we write out

$$|\psi_j\rangle = a_j |n, e_1, e_2\rangle + b_j |n + 1, e_1, g_2\rangle + c_j |n + 1, g_1, e_2\rangle + d_j |n + 2, g_1, g_2\rangle. \quad (3)$$

The ground-state energy as a function of the coupling strength λ and the detuning Δ is shown in Figure 1 (Δ and λ are measured in units of J), from which we can see that both the coupling strength λ and the detuning Δ can control the ground-state energy as well as the Berry phase as will be shown. The level crossings lead to the presence of non-analyticity in the energy spectrum and the corresponding quantum phase transition (QPT) [15] can be represented by the discontinuous variation of the ground state Berry phase.

It is known that in the standard semiclassical framework the field operators $a$and $a^\dagger$ are replaced by the classical amplitude with rotation factors $e^{-i\varphi(t)}$ and $e^{i\varphi(t)}$ with $\varphi(t) = \omega_0 t$. Therefore, the semiclassical Hamiltonian corresponding to Hamiltonian (1) can be written as

$$H = \Delta (\sigma_1^z + \sigma_2^z) + \lambda\alpha \left[e^{-i\varphi(t)} \left(\sigma_1^+ + \sigma_2^+\right) + e^{i\varphi(t)} \left(\sigma_1^- + \sigma_2^-\right)\right] + J \left(\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+\right) \quad (4)$$

where α is the amplitude of the oscillating field. It can be seen easily that this semiclassical Hamiltonian can be expressed in terms of an effective vector field $B = (2\lambda\alpha \cos\varphi, 2\lambda\alpha \sin\varphi, \Delta)$ as

$$H = B (\sigma_1 + \sigma_2) + J \left(\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+\right). \quad (5)$$

When $\varphi(t) = \omega_0 t$ is changed adiabatically and periodically in the parameter space of the effective vector field B, the semiclassical Berry phase can also be obtained [16]. However, if the controlled external field is quantized, this effective vector field becomes part of the system itself and therefore cannot be taken into account any longer as an external variable. But the corresponding state can also be manipulated in the parameter space of the coupling strength λ and detuning Δ for Hamiltonian (1). Following the spirit of [11], this Berry phase can be evaluated by introducing the following phase shift:

$$R(t) = e^{-i\varphi(t)a^\dagger a} \quad (6)$$

where $\varphi(t) = \omega_0 t$ should be changed adiabatically and periodically.

This phase shift $R(t)$ can lead to the time-dependent transformation $|\Psi_j(t)\rangle = R(t)|\psi_j\rangle$ or $|\psi_j\rangle = R^\dagger(t)|\Psi_j(t)\rangle$, where $|\psi_j\rangle$ is the eigenvector of the time-independent eigen equation $H|\psi_j\rangle = E_j|\psi_j\rangle$ and $|\Psi_j(t)\rangle$ is the eigenvector of the time-dependent eigen equation $id|\Psi_j(t)\rangle/dt = H'(t)|\Psi_j(t)\rangle$ with $H'(t) = R(t)HR^\dagger(t) - iR(t)dR^\dagger(t)/dt$. For the time-dependent eigen equation $id|\Psi_j(t)\rangle/dt = H'(t)|\Psi_j(t)\rangle$ the Berry phase can be calculated in terms of standard definition using

$$\gamma_j = i\int_0^T \langle\Psi_j(t)| \frac{d}{dt} |\Psi_j(t)\rangle\, dt = i\int_0^{2\pi} d\langle\Psi_j(\varphi)| \frac{d}{d\varphi} |\Psi_j(\varphi)\rangle\, d\varphi. \quad (7)$$

By using the transformations $|\Psi_j(t)\rangle = R(t)|\psi_j\rangle$ and $\langle\Psi_j(t)| = \langle\psi_j| R^\dagger(t)$, the final Berry phase corresponding to Hamiltonian (1) can be given as

$$\gamma_j = i\int_0^{2\pi} \langle\psi_j| R^\dagger(\varphi)\frac{d}{d\varphi}R(\varphi)|\psi_j\rangle\, d\varphi. \quad (8)$$

For the Jaynes-Cummings model whose Hamiltonian reads $\omega a^\dagger a + \frac{\omega_0}{2}\sigma^z + \lambda\left(a\sigma^+ + a^\dagger\sigma^-\right)$, the Berry phase can be evaluated as $\gamma_+ = \pi(1 - \cos\theta_n) + 2\pi n$ and $\gamma_- = -\pi(1 - \cos\theta_n) + 2\pi(n+1)$ with $\cos\theta_n = \frac{\Delta}{\sqrt{\Delta^2 + 4\lambda^2(n+1)}}$, which can be mapped into the semiclassical results in the coherent state representation with large amplitude [11]. However, for Hamiltonian (1) the Berry phase can be derived from equations (3) as the following:

$$\gamma_j = 2\pi\left[n|a_j|^2 + (n+1)\left(|b_j|^2 + |c_j|^2\right) + (n+2)|d_j|^2\right]. \quad (9)$$

This is very interesting for discussions on the ground-state Berry phase ($n = 0$). A novel observation of this paper is that the ground-state Berry phase can be controlled by both the coupling strength λ and the detuning Δ shown in Figure 2. The behavior of the ground-state Berry phase is similar to that of the ground-state concurrence as a measure of the entanglement between the two qubits [14]. There are a series of critical points of detuning for different coupling strength that the ground-state Berry phase take place mutation, no less than ground-state concurrence. However, the ground-state concurrence C jump up with the increase of absolute value of detuning, whether detuning is positive or negative, whereas the ground-state

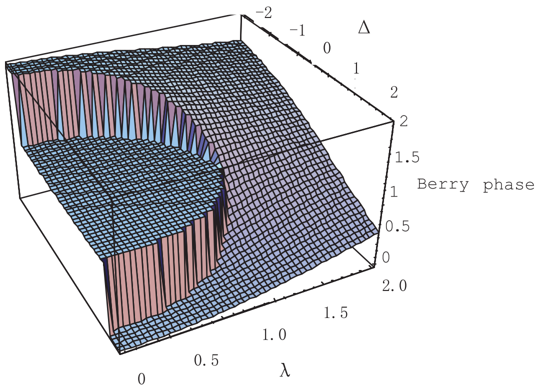

Fig. 2. The ground-state Berry phase versus the coupling strength λ and the cavity detuning Δ. Δ and λ are measured in units of J.

Berry phase jump down when detuning is negative. This result extends the conclusion of the reference [18], in which the concurrence is related to the cyclic geometric phase of the individual spins.

In conclusion, the Berry phase in Tavis-Cummings model with the dipole-dipole interaction has been obtained. A novel feature of the Berry phase can be controlled by the coupling strength λ and the frequency detuning Δ of the quantized cavity field, which has important application in geometric quantum computing.

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